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Noncommutative QED and anomalous dipole moments

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Abstract: We study QED on noncommutative spaces, NCQED. In particular we present the detailed calculation for the noncommutative electron-photon vertex and show that the Ward identity is satisfied. We discuss that in the noncommutative case moving electron will show *electric* dipole effects. In addition, we work out the electric and magnetic dipole moments up to one loop level. For the magnetic moment we show that noncommutative electron has an intrinsic (spin independent) magnetic moment.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Renormalization Regularization and Renormalons.

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Contents

1. Introduction

Although, physics on the noncommutative spaces has a long standing story [1], recently it has been re-motivated by string theory arguments [2]. Apart from the string theory interests the field theories on such spaces, noncommutative field theories (NCFT's), in their own turn are very interesting. Although being non-local, it has been argued that they can be treated as sensible field theories, and in the last two years there have been a lot of work devoted to the study of NCFT's. The question of renormalizability of NCFT's in general and in particular NC scalar and NC Yang-Mills (NCYM) theories have been addressed extensively [4]–[13].[1]

For the NC scalar theories it has been shown that real Φ^4 theory in 4 dimensions is two loop renormalizable [5, 13]. The pure NCYM have been studied only in one loop level and shown to be renormalizable. The β-function of these theories (pure NCYM) is found to be the same as the corresponding commutative theory.

[1]Because of the huge number of papers on this issue, hereby we apologize for all the related works which have not been quoted.

JHEP08(2000)045

The problem of adding fermions (matter fields) has not been studied in detail. In particular, noncommutative version of QED, NCQED, has been discussed in [8, 10, 11, 14, 15]. We emphasize that here we only consider spaces-like noncommutativities and not the time-like one (noncommutative space-times). In the latter case, it has been shown that the corresponding field theories are not unitary [16]. However, for the light-like noncommutativity it has been shown that we still have a well-defined quantum theory [17].

In this work we study some details of NCQED. Working out the electron-photon interaction vertex in NCQED up to one loop, we find the so-called vertex functions and thereby we read off the anomalous magnetic moment. By explicit calculation we show that Ward identity is satisfied in the NCQED case. Since we have an extra suitable vector in our theory, there is some room for new type of magnetic moment which is spin independent and is proportional to the noncommutativity parameter.

As it has been discussed in [11, 18] particles in the noncommutative gauge theories show *electric dipole* effects. We recall this property in the classical case, and also study one loop quantum corrections to the electric dipole moment.

The paper is organized as following. In section 2, we briefly review the preliminaries we need; we introduce noncommutative spaces and field theories in general and specify the NCQED by presenting its classical action and basic Feynman graphs. Then in section 3, we work out details of noncommutative electron-photon vertex at one loop level. In addition, renormalizing the corresponding graphs in the minimal subtraction (MS) scheme, we discuss the IR/UV mixing which is a characteristic of any NCFT. In section 4, using the renormalized vertex functions we find "electromagnetic dipole" properties of noncommutative electron. We end this work by conclusions and remarks.

2. Preliminaries

2.1 Noncommutative spaces

Usual quantum mechanics is formulated on commutative spaces satisfying the following commutation relations,

$$[\hat{X}_i, \hat{P}_j] = i\hbar\delta_{ij}\,, \qquad [\hat{X}_i, \hat{X}_j] = 0 \qquad \text{and} \qquad [\hat{P}_i, \hat{P}_j] = 0\,. \tag{2.1}$$

Then in order to describe a noncommutative space, the above commutation relations should be changed as,

$$[\hat{X}_i, \hat{P}_j] = i\hbar\delta_{ij}\,, \qquad [\hat{X}_i, \hat{X}_j] = i\theta_{ij} \qquad \text{and} \qquad [\hat{P}_i, \hat{P}_j] = 0\,, \tag{2.2}$$

where θ_{ij}, is the noncommutative *constant* of dimension $[M]^{-2}$. We can see that for theories on such spaces the Lorentz symmetry is explicitly violated however, it will be recovered in the $\theta \to 0$ limit. The above noncommutative theory can be expanded to include noncommuting space-time, i.e.

$$[\hat{X}_\mu, \hat{X}_\nu] = i\theta_{\mu\nu}\,. \tag{2.3}$$

These type of theories were studied in [16] and shown to suffer from the loss of unitarity, so for the purpose of our work here, we will only consider the noncommuting spaces. Since our field theory is better formulated through the path integral formulation, we can implement the noncommutivity of space into path integral formulation through what is known as the Weyl-Moyal correspondence [12],[2]

$$\begin{aligned}
\hat{\Phi}(\hat{X}) &\longleftrightarrow \Phi(x)\,; \\
\hat{\Phi}(\hat{X}) &= \int_\alpha e^{i\alpha\hat{X}}\phi(\alpha)d\alpha\,, \\
\phi(\alpha) &= \int e^{-i\alpha x}\Phi(x)dx\,,
\end{aligned} \tag{2.4}$$

where α and x are real variables. Then,

$$\begin{aligned}
\hat{\Phi}_1(\hat{X})\hat{\Phi}_2(\hat{X}) &= \iint\limits_{\alpha\beta} e^{i\alpha\hat{X}}\phi(\alpha)e^{i\beta\hat{X}}\phi(\beta)d\alpha d\beta \\
&= \iint\limits_{\alpha\beta} e^{i(\alpha+\beta)\hat{X}-\frac{1}{2}\alpha_\mu\beta_\nu[\hat{X}_\mu,\hat{X}_\nu]}\phi(\alpha)\phi(\beta)d\alpha d\beta\,,
\end{aligned} \tag{2.5}$$

hence

$$\begin{aligned}
\hat{\Phi}_1(\hat{X})\hat{\Phi}_2(\hat{X}) &\longleftrightarrow \left(\Phi_1 * \Phi_2\right)(x)\,, \\
\left(\Phi_1 * \Phi_2\right)(x) &\equiv \left[e^{\frac{i}{2}\theta_{\mu\nu}\partial_{\zeta\mu}\partial_{\eta\nu}}\Phi(x+\zeta)\Phi(x+\eta)\right]_{\zeta=\eta=0}\,.
\end{aligned} \tag{2.6}$$

In other words, the noncommutative version of a field theory is obtained by replacing all the field products by the star product (2.6). It is easy to check that the Moyal bracket of coordinates defined by,

$$[x_\mu, x_\nu]_{MB} = x_\mu * x_\nu - x_\nu * x_\mu\,, \tag{2.7}$$

satisfies the commutation relations on the noncommutative spaces (2.3).

2.2 The noncommutative QED

In this part we introduce the structure of the action for NCQED [8]. We will notice that due to the presence of the star product and the Moyal brackets, the noncommutative U(1) is similar to non-abelian gauge theories. As usual, the action for a

[2]In the following equations there are some obvious factors of 2π which we avoid them here.

gauge theory consists of two parts, the gauge fields and matter fields, fermions. To start with, we write the term for the gauge fields:

$$S_{YM} = -\int d^4x \frac{1}{4e^2} F_{\mu\nu} * F^{\mu\nu}\,, \tag{2.8}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_{MB}\,. \tag{2.9}$$

The above action enjoys the noncommutative U(1) symmetry, defined by

$$A_\mu \to A'_\mu(x) = U(x) * A_\mu * U(x)^{-1} + iU(x) * \partial_\mu U(x)^{-1}\,, \tag{2.10}$$

$$U(x) = \exp *(i\lambda) \equiv 1 + i\lambda - \frac{1}{2}\lambda * \lambda + \cdots\,. \tag{2.11}$$

Under the above transformation, the field strength $F_{\mu\nu}$, transforms as

$$F_{\mu\nu} \to F'_{\mu\nu} = U(x) * F_{\mu\nu} * U(x)^{-1}\,, \tag{2.12}$$

and hence

$$S_{YM} \to S'_{YM} = -\int d^4x \frac{1}{4e^2} U(x) * F_{\mu\nu} * U(x)^{-1} * U(x) * F^{\mu\nu} * U(x)^{-1}\,. \tag{2.13}$$

using,

$$U(x) * U(x)^{-1} = U(x)^{-1} * U(x) = 1\,,$$

and the cyclic property of the star product under the integral (see appendix), we conclude that (2.8) is invariant under the above defined gauge transformation. Also using the property that under the integral the star product is commuting, one can remove the star product between two F's, then

$$S_{YM} = -\int d^4x \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}\,. \tag{2.14}$$

In order to add the matter fields, we need the notion of covariant derivative, D_μ, for which we demand that our fermionic sector is also invariant under the above mentioned noncommutative local gauge transformations. We notice that one can extend the local gauge transformations to fermions in the following two different ways,

$$\Psi(x) \to \Psi'(x) = U(x) * \Psi(x)\,, \tag{2.15}$$

or

$$\tilde{\Psi}(x) \to \tilde{\Psi}'(x) = \tilde{\Psi} * U(x)^{-1}\,. \tag{2.16}$$

Therefore, one can define two covariant derivatives corresponding to two different fermions:

$$D_\mu \Psi = \partial_\mu \Psi - iA_\mu * \Psi\,, \tag{2.17}$$

or

$$D_\mu \tilde{\Psi} = \partial_\mu \tilde{\Psi} + i\tilde{\Psi} * A_\mu \, . \tag{2.18}$$

From the above two equations we see that by taking the limit, $\theta_{\mu\nu} \to 0$, the star product disappears and the two fermion fields $\Psi(x)$ and $\tilde{\Psi}(x)$ will show different charges. In fact it has been shown that these two fermions are related by the noncommutative version of the charge conjugation [14].

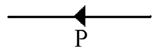
So, altogether the full lagrangian can be written as,

$$S = \int d^4x \left(-\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(\not{D} - m)\Psi + L_{\text{gauge}} + L_{\text{ghost}} \right) , \tag{2.19}$$

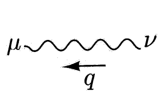
where we have also added the term L_{gauge} for gauge fixing. Since the quadratic terms are the same as in the usual QED, we have the same gauge fixing term. The L_{ghost} accounts for the ghost fields appearing in the gauge fixing procedure, similar to that of the non-abelian gauge theories.

2.3 Feynman rules for NCQED

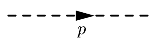
Using the property of the star product under the integral sign and that, the quadratic terms are not changed in the presence of the star product, we conclude that the propagators for the free fermion, gauge and the ghost fields are the same as in the case of usual QED, i.e.



$$= \frac{i}{\not{p} - m + i\epsilon} \, , \tag{2.20}$$



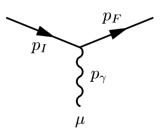
$$= \frac{g^{\mu\nu}}{i(p^2 + i\epsilon)} \, , \tag{2.21}$$



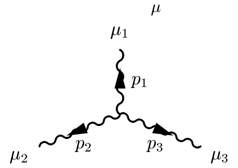
$$= \frac{1}{i(p^2 + i\epsilon)} \, , \tag{2.22}$$

where we were using the Feynman gauge.

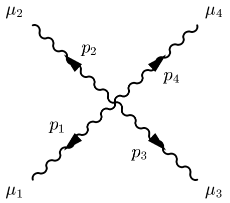
For the interaction terms, we see that they are similar to those of non-abelian gauge theories [4, 8], in the sense that we get cubic and quadric interaction vertices for the gauge fields besides the usual vertices found usual QED. Here we just present the Feynman rules, to show their similarities and differences with the non-abelian case.



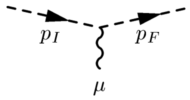
$$= ie\gamma^\mu e^{\frac{i}{2} p_I \times p_F}$$



$$= -2e \sin\left(\frac{1}{2} p_1 \times p_2\right) \Big[(p_1 - p_2)^{\mu_3} g^{\mu_1\mu_2} + (p_2 - p_3)^{\mu_1} g^{\mu_2\mu_3} + \\ + (p_3 - p_1)^{\mu_2} g^{\mu_3\mu_1} \Big]$$



$$= -4ie^2\left[(g^{\mu_1\mu_3}g^{\mu_2\mu_4} - g^{\mu_1\mu_4}g^{\mu_2\mu_3})\sin\left(\frac{1}{2}p_1\times p_2\right)\sin\left(\frac{1}{2}p_3\times p_4\right)\right]\times$$
$$\times\left[(g^{\mu_1\mu_4}g^{\mu_2\mu_3} - g^{\mu_1\mu_2}g^{\mu_3\mu_4})\sin\left(\frac{1}{2}p_3\times p_1\right)\sin\left(\frac{1}{2}p_2\times p_4\right)\right]\times$$
$$\times\left[(g^{\mu_1\mu_2}g^{\mu_3\mu_4} - g^{\mu_1\mu_3}g^{\mu_2\mu_4})\sin\left(\frac{1}{2}p_1\times p_4\right)\sin\left(\frac{1}{2}p_2\times p_3\right)\right]$$



$$= 2iep_F^\mu \sin\left(\frac{1}{2}p_I \times p_F\right). \tag{2.23}$$

We observe that all vertices here are similar to those in non-abelian gauge theories in which the structure constant is replaced by $2\sin(\frac{1}{2}p\times p')$. This can be seen if we notice that the structure constants appear because of the commutation $[A_a, A_b] = if_{abc}A^c$ in non-abelian theories. Hence we expect the appearance of the factor $2\sin(\frac{1}{2}p \times p')$ as a consequence of the Moyal bracket, i.e.

$$\begin{aligned}
[A_\mu, A_\nu]_{MB} &= A_\mu * A_\nu - A_\nu * A_\mu \\
&= \int d^4p\ d^4p'\ A_\mu(p)A_\nu(p')\left(e^{\frac{i}{2}p\times p'} - e^{\frac{-i}{2}p\times p'}\right)e^{i(p+p')\cdot x} \\
&= \int d^4p\ d^4p'\ 2iA_\mu(p)A_\nu(p')\sin\left(\frac{1}{2}p\times p'\right)e^{i(p+p')\cdot x}.
\end{aligned} \tag{2.24}$$

3. Electron-photon vertex at one loop level

In the previous section we introduced the NCQED, and showed that in this theory there are new type of vertices similar to those found in non-abelian gauge theories. In this section we perform explicit calculation of the vertex function for the photon-electron at the one loop level which are expected to contribute to the anomalous magnetic moment.

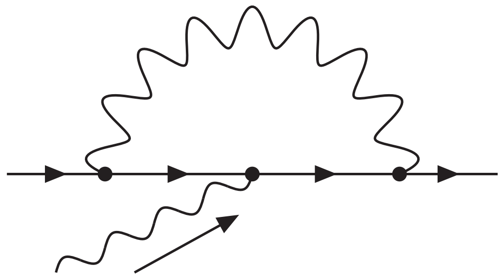
3.1 Vertex structure at the one loop level

In the case of NCQED and because of the three photon vertices, the electron-photon vertex receives contributions from the two diagrams of figure 1.

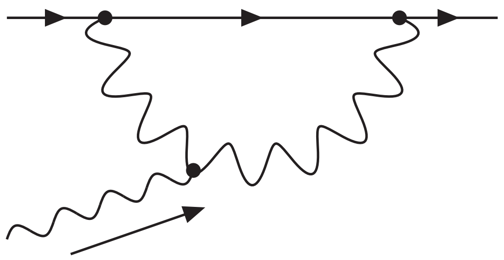
The first diagram is similar to, what can also be found in the usual QED however, the second diagram is completely new. Now we proceed to write the analaytic expression of the first diagram,

$$\begin{aligned}
\Gamma_\mu^{(a)} = i(-ie)^2 e^{\frac{i}{2}p\times p'}\int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot\tilde{q}}\frac{\gamma_\sigma}{k^2 - m_\gamma^2 + i\epsilon}\frac{\not{p}' - \not{k} + m}{(p'-k)^2 - m^2 + i\epsilon}\gamma_\mu \times \\
\times \frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2 + i\epsilon}\gamma^\sigma,
\end{aligned} \tag{3.1}$$

where m_γ is a photon mass accounting for the IR divergences.



(a) One loop QED-like correction to $\psi\bar{\psi}A_\mu$



(b) Nonabelian-type correction to $\psi\bar{\psi}A_\mu$

Figure 1: One loop correction to $\psi\bar{\psi}A_\mu$ vertex.

Using the on mass shell conditions the numerator in the above expression may be written as

$$4\left\{\gamma_\mu\left[(p'-k)(p-k)-\frac{k^2}{2}\right]+\not{k}(p'+p-k)_\mu - mk_\mu\right\}. \tag{3.2}$$

To compute the above integral, we replace the propagators by the Schwinger parameters:

$$\frac{i}{p^2-m^2+i\epsilon}=\int_0^\infty d\alpha e^{i\alpha(p^2-m^2+i\epsilon)} \tag{3.3}$$

and use the auxiliary integral,

$$\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(z-\tilde{q})}\frac{1}{(k^2-m_\gamma^2)(k^2-2p'\cdot k)(k^2-2p\cdot k)}= \tag{3.4}$$

$$=\frac{1}{(4\pi)^2}\int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3\frac{\exp\left[\frac{-i}{a}\left((\alpha_1+\alpha_2+\alpha_3)\alpha_1 m_\gamma^2+(\frac{(z-\tilde{q})}{2}-\alpha_2 p'-\alpha_3 p)^2\right)\right]}{(\alpha_1+\alpha_2+\alpha_3)^2},$$

where

$$\tilde{q}_\nu=\theta_{\mu\nu}q^\mu\,, \tag{3.5}$$

and $a=\alpha_1+\alpha_2+\alpha_3$. The introduction of the factor $e^{ik\cdot z}$ in the integrand allows us to obtain the required expression in the numerator of Γ_μ by differentiation [21].

After symmetrization in α_2 and α_3 we obtain

$$\begin{aligned}\Gamma_\mu^{(a)} &= \frac{\alpha}{i\pi}e^{\frac{i}{2}p\times p'}\int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3 \frac{e^{i/a\Lambda_{\text{eff}}^2}}{a^3}\exp\left[\frac{-i}{a}(\alpha_1 a m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3 q^2)\right]\times\\ &\times e^{\frac{-i(\alpha_2+\alpha_3)p\tilde{q}}{a}}\left\{\gamma_\mu\left[ap'\cdot p-\frac{(\alpha_2+\alpha_3)(p+p')^2}{2}+\frac{i}{2}+\frac{(m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2)}{2a}\right]+\right.\\ &\qquad+\frac{m}{2a}(p'+p)_\mu\alpha_1(\alpha_2+\alpha_3)-\frac{\gamma_\mu}{4a}\left((p'+p)(\alpha_1+\alpha_1+\alpha_2+\alpha_3)-\frac{1}{2}\tilde{q}\right)\cdot\tilde{q}+\\ &\qquad\left.+\frac{2m(\alpha_1+2\alpha_2+2\alpha_3)\tilde{q}_\mu+((p'+p)(2\alpha_1+\alpha_2+\alpha_3)-\tilde{q})_\mu\gamma\cdot\tilde{q}}{4a}\right\}, \end{aligned}\tag{3.6}$$

where we have inserted a UV regulator, $\exp(i/a\Lambda^2)$, with the following notation,[3]

$$\Lambda_{\text{eff}}^2 = \frac{1}{\Lambda^{-2}-\frac{\tilde{q}\cdot\tilde{q}}{4}}\,. \tag{3.7}$$

After the Wick rotation, i.e. $\alpha_i \to \alpha_i/i$, with the help of the following identity,

$$1 = \int_0^\infty d\rho\,\delta(\rho-\sum\alpha_i)\,, \tag{3.8}$$

and rescaling $\alpha_i \to \rho\alpha_i$, (3.6) is obtained to be

$$\begin{aligned}\Gamma_\mu^{(a)} &= \frac{-\alpha}{\pi}e^{\frac{i}{2}p\times p'}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\delta\left(1-\sum\alpha_i\right)e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}\times\left(A_\mu+\frac{B_\mu}{\rho}+\frac{C_\mu}{\rho^2}\right)\times\\ &\times\int_0^\infty d\rho\exp\left(-\rho(\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2)-\frac{1}{\rho\Lambda_{\text{eff}}^2}\right)\,,\end{aligned}\tag{3.9}$$

where,

$$\begin{aligned}A_\mu &= \gamma_\mu\left[p'.p-\frac{(\alpha_2+\alpha_3)(p'+p)^2)}{2}+\frac{1}{2}m^2(\alpha_2+\alpha_3)^2-\frac{1}{2}\alpha_2\alpha_3q^2\right]+\\ &\quad+\frac{m}{2}\alpha_1(\alpha_2+\alpha_3)(p'+p)_\mu\,,\\ iB_\mu &= \frac{\gamma_\mu}{2i}+\frac{\gamma_\mu p\cdot\tilde{q}}{2}(2-\alpha_2-\alpha_3)+\frac{m}{2}(1+\alpha_2+\alpha_3)\tilde{q}_\mu+\frac{1}{4}(\alpha_1+1)(p'+p)_\mu\gamma\cdot\tilde{q}\,,\\ C_\mu &= \frac{-\gamma_\mu\tilde{q}\cdot\tilde{q}}{8}+\frac{\tilde{q}_\mu\gamma\cdot\tilde{q}}{4}\,.\end{aligned}\tag{3.10}$$

Performing the integral over ρ, we get

$$\begin{aligned}\Gamma_\mu^{(a)} &= \frac{-\alpha}{\pi}e^{\frac{i}{2}p\times p'}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\delta\left(1-\sum\alpha_i\right)e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}\times\\ &\times\left(\frac{2A_\mu K_1(2\sqrt{X})}{\sqrt{X}\Lambda_{\text{eff}}^2}+2B_\mu K_0(2\sqrt{X})+2\sqrt{X}\Lambda_{\text{eff}}^2C_\mu K_1(2\sqrt{X})\right),\end{aligned}\tag{3.11}$$

[3] We should note that due to anti-symmetry of $\theta_{\mu\nu}$, $\tilde{q}\cdot\tilde{q}$ is negative valued.

JHEP08(2000)045

where

$$X \equiv \frac{\alpha_1 m_\gamma^2 + (\alpha_2+\alpha_3)^2 m^2 - \alpha_2\alpha_3 q^2}{\Lambda_{\rm eff}^2}\,,$$

and K_0, K_1 are the modified Bessel functions of the first and second type respectively.

Now we consider the contribution from the second diagram, the analytic expression of which reads as

$$\Gamma_b^\mu = -ie^2 e^{\frac{i}{2}p\times p'} \int \frac{d^4k}{(2\pi)^4} \frac{\left(1-e^{ik\cdot\tilde{q}}e^{ip'\cdot\tilde{p}}\right)}{(k^2-m^2)((p'-k)^2-m_\gamma^2)((p-k)^2-m_\gamma^2)} \times$$
$$\times\left\{\gamma_\nu(\not{k}+m)\gamma_\rho\left[g^{\mu\nu}(2p'-p-k)^\rho + g^{\nu\rho}(2k-p'-p)^\mu + g^{\rho\mu}(2p-p'-k)^\nu\right]\right\}. \tag{3.12}$$

Using the gamma matrices algebra and the on shell condition the numerator can be written as,

$$8mk_\mu - 2\not{k}(p'+p+2k)_\mu + 2\gamma_\mu(2p'\cdot k + 2p\cdot k - k^2 - 3m^2)\,. \tag{3.13}$$

At this point we notice that (3.13) can be separated into two parts, one containing the phase $e^{-ik\cdot\tilde{q}}$, and the other not, i.e. $\Gamma_\mu^b = \Gamma_\mu^{b1} + \Gamma_\mu^{b2}$. First we perform the part containing the phase, $\Gamma_\mu^{(b2)}$, then the other term, $\Gamma_\mu^{(b1)}$, can be recovered easily.

Following the same spirit of the previous calculation, the auxiliary integral is introduced and then doing the gaussian integral,

$$\int \frac{d^4k}{(2\pi)^4} \frac{e^{ik.(z+\tilde{q})}}{(k^2-m^2)((p'-k)^2-m_\gamma^2)((p-k)^2-m_\gamma^2)} =$$
$$= \frac{1}{4\pi^2}\int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3 \frac{1}{a^2} e^{\frac{-i}{a}\left(\frac{z^2}{4}-z.(\alpha_2 p'+\alpha_3 p-\frac{\tilde{q}}{2})\right)} e^{\frac{i}{a}(\alpha_2+\alpha_3)p\cdot\tilde{q}} \times$$
$$\times \exp\left[-i\left(m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+\frac{1}{a}(m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3 q^2)-\frac{1}{a\Lambda_{\rm eff}^2}\right)\right], \tag{3.14}$$

Producing the numerator through the derivatives over the auxiliary field, a Wick rotation, rescaling the α_i's and finally inserting the identity $1 = \int_0^\infty \frac{d\rho}{\rho}\delta(1-\sum\alpha_i)$, the expression $\Gamma_\mu^{(b2)}$ is found to be

$$\Gamma_\mu^{(b2)} = \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right) e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}} e^{-ip\times p'} \times \int d\rho$$
$$\exp\left[-\rho\left(m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3 q^2\right) - \frac{1}{\rho\Lambda_{\rm eff}^2}\right] \times$$
$$\times\left(\tilde{A}_\mu + \frac{\tilde{B}_\mu}{\rho} + \frac{\tilde{C}_\mu}{\rho^2}\right), \tag{3.15}$$

where now, $\tilde{A}_\mu, \tilde{B}_\mu$ and $\tilde{C}_\mu$ are,

$$
\begin{aligned}
\tilde{A}^\mu &= \frac{1}{2}\gamma_\mu \left[(\alpha_2+\alpha_3)(p'+p)^2 - 3m^2 - m^2(\alpha_2+\alpha_3)^2 + \alpha_2\alpha_3 q^2\right] + \\
&\quad + \frac{m\alpha_1}{2}(\alpha_2+\alpha_3)(p'+p)_\mu \,, \\
i\tilde{B}_\mu &= \frac{3i}{2}\gamma_\mu + \frac{1}{2}(m\tilde{q}_\mu + \gamma_\mu p\cdot\tilde{q})(2-\alpha_2-\alpha_3) - \frac{\gamma\cdot\tilde{q}}{4}(p'+p)_\mu(1+\alpha_2+\alpha_3)\,, \\
\tilde{C}_\mu &= \frac{\gamma\cdot\tilde{q}\tilde{q}_\mu}{4} + \frac{\gamma_\mu \tilde{q}\cdot\tilde{q}}{8}\,.
\end{aligned}
\tag{3.16}
$$

Integration over ρ leads to

$$
\begin{aligned}
\Gamma_\mu^{(b2)} &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right) e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}} e^{ip'\cdot p} \times \\
&\quad \times \left(\frac{2\tilde{A}_\mu K_1(2\sqrt{Y})}{\sqrt{Y}\Lambda_{\text{eff}}^2} + 2\tilde{B}_\mu K_0(\sqrt{Y}) + \tilde{C}_\mu\sqrt{Y}\Lambda_{\text{eff}}^2 K_1(2\sqrt{Y})\right),
\end{aligned}
\tag{3.17}
$$

with

$$
Y \equiv \frac{m^2(\alpha_1-\alpha_2-\alpha_3) + m_\gamma^2(\alpha_2+\alpha_3) + m^2(\alpha_2+\alpha_3)^2 - \alpha_2\alpha_3 q^2}{\Lambda_{\text{eff}}^2}\,. \tag{3.18}
$$

The $\Gamma_\mu^{(b1)}$ term, can be easily recovered from the above expression by setting any term proportional to $\theta^{\mu\nu}$ (i.e. $\tilde{q}^\mu$) to zero. Then we have,

$$
\Gamma_\mu^{(b1)} = \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)\left(\frac{2\bar{A}_\mu K_1(2\sqrt{Z})}{\sqrt{Z}\Lambda^2} + 2\bar{B}_\mu K_0(2\sqrt{Z})\right), \tag{3.19}
$$

while Z is now,

$$
Z \equiv \frac{m^2(\alpha_1-\alpha_2-\alpha_3) + m_\gamma^2(\alpha_2+\alpha_3) + m^2(\alpha_2+\alpha_3)^2 - \alpha_2\alpha_3 q^2}{\Lambda^2}\,. \tag{3.20}
$$

and,

$$
\begin{aligned}
\bar{A}^\mu &= \frac{1}{2}\gamma_\mu \left((\alpha_2+\alpha_3)(p'+p)^2 - 3m^2 - m^2(\alpha_2+\alpha_3)^2 + \alpha_2\alpha_3 q^2\right) + \\
&\quad + \frac{m\alpha_1}{2}(\alpha_2+\alpha_3)(p'+p)_\mu \\
\bar{B}_\mu &= \frac{3}{2}\gamma_\mu\,.
\end{aligned}
\tag{3.21}
$$

3.2 Renormalization

Now we try to look at the divergences appearing in our diagrams. In the case of usual QED we have a logarithmic UV divergence, and the problem of IR divergence was fixed by introducing the photon a finite mass, m_γ. Looking at our expressions of the form factors , we see that they contain K_0, and K_1, and both of the functions contain either $1/\Lambda_{\rm eff}^2$ or $1/\Lambda^2$ in their arguments. Taking the high energy limit $\Lambda^2 \to \infty$, or the low energy limit $q \to 0$ simultaneously we see that all terms containing K_1 are finite, but there appear to be a logarithmic divergence due to K_0. Therefore, we recover the same logarithmic divergence of usual QED when taking the IR limit. The noncommutative QED was shown to be renormalizable up to the one loop level by adding the relevant counter terms [8], so we can safely drop the singular parts in K_0, and keeping only the finite parts. Now taking the $\Lambda \to \infty$ limit, and dropping the divergent parts the renormalized Gamma functions can be written as,

$$\begin{aligned}
\Gamma^{(a)}_{\mu({\rm UV-ren})} &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right) e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}} \times \\
&\times\left(\frac{A_\mu}{\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2} - 2B_\mu\gamma_{\rm Euler}+\Lambda_{\rm eff}^2 C_\mu\right), \quad (3.22)\\
\Gamma^{(b1)}_{\mu\ ({\rm UV-ren})} &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right) \times \\
&\times\left(\frac{\bar{A}_\mu}{m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2} - \right.\\
&\left. - 2\bar{B}_\mu\gamma_{\rm Euler}\right), \quad (3.23)
\end{aligned}$$

and

$$\begin{aligned}
\Gamma^{(b2)}_{\mu\ ({\rm UV-ren})} &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right) e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'} \times \\
&\times\left(\frac{\tilde{A}_\mu}{m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2} - \right.\\
&\left. - 2\tilde{B}_\mu\gamma_{\rm Euler}+\tilde{C}_\mu\Lambda_{\rm eff}^2\right). \quad (3.24)
\end{aligned}$$

Finally the full renormalized Gamma function for the electron-photon vertex at the one loop level is

$$\begin{aligned}
\Gamma_{\mu(UV-ren)} &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right) \times \\
&\times\left(\frac{A_\mu e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}}{\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2} + \right.\\
&+\frac{\bar{A}_\mu(1-e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'})}{m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2} - \\
&-2\gamma_{\rm Euler}(B_\mu e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}+\bar{B}_\mu-\tilde{B}_\mu e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'}) + \\
&\left.+\Lambda_{\rm eff}^2(C_\mu e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}-\tilde{C}_\mu e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'})\right). \quad (3.25)
\end{aligned}$$

In equation (3.25), we have given the UV-renormalized gamma function, but we can see that in the last term we still keep the cut-off in the expression, and this can be understood by the UV/IR mixing. In order this we will discuss both limits applied to this term, first considering the UV limit, i.e. $1/\Lambda^2 \ll \tilde{q}\cdot\tilde{q}$ ($\Lambda^2_{\rm eff} \sim 1/\tilde{q}\cdot\tilde{q}$), we see that the term is finite, but when taking the IR limit first, i.e. $1/\Lambda^2 \gg \tilde{q}\cdot\tilde{q}$ then $\Lambda^2_{\rm eff} \sim 1/\Lambda^2$, so it may seem that this term will lead to an IR divergence. However, C_μ terms contain two type of terms, both proportional to $\tilde{q}^2$, and since $\Lambda^2\tilde{q}^2 \ll 1$, in the IR limit, this term will be totally irrelevant. One should note that order of taking the $\Lambda^2 \to \infty$ and $q \to 0$ is very important in our arguments; this is a generic feature of NCFT's and is called IR/UV mixing [6, 7]. Hence the fully renormalized vertex function is,

$$\begin{aligned}
\Gamma_{\mu(ren)} &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)\times \\
&\times\Bigg(\frac{A_\mu e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}}{\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2 m^2-\alpha_2\alpha_3 q^2}+ \\
&\quad+\frac{\bar{A}_\mu(1-e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'})}{m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3 q^2}- \\
&\quad-2\gamma_{\rm Euler}(B_\mu e^{-i(\alpha_2+\alpha_3)\cdot\tilde{q}}+\bar{B}_\mu-\tilde{B}_\mu e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'})\Bigg).
\end{aligned} \tag{3.26}$$

For the moment we can see that Γ^μ can be written in the form,

$$\Gamma^\mu = E_1\gamma^\mu + H_1(p'+p)^\mu + G_1\tilde{q}^\mu + E_2\gamma^\mu p\cdot\tilde{q} + H_3(p'+p)^\mu\gamma\cdot\tilde{q}. \tag{3.27}$$

Collecting the different cofficients of γ^μ, $(p'+p)^\mu$, $\tilde{q}^\mu$, $\gamma^\mu p\cdot\tilde{q}^\mu$, $(p'+p)^\mu\gamma\cdot\tilde{q}$, we have

$$\begin{aligned}
E_1 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\delta\left(1-\sum\alpha_i\right)\times\left(1-e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'}\right)\times \\
&\times\Bigg\{\left[\frac{(2p'\cdot p-(\alpha_2+\alpha_3)(p'+p)^2)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2)}{2(\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2)}+\gamma_{\rm Euler}\right]e^{-i(\alpha_2+\alpha_3)p.\tilde{q}} \\
&+\left[\frac{((\alpha_2+\alpha_3)(p'+p)^2-3m^2-m^2(\alpha_2+\alpha_3)^2+\alpha_2\alpha_3q^2)}{m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2}-\frac{3\gamma_{\rm Euler}}{2}\right]\Bigg\},
\end{aligned}$$

$$\begin{aligned}
H_1 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\delta\left(1-\sum\alpha_i\right)\times \\
&\times\Bigg\{\frac{m\alpha_1(\alpha_2+\alpha_3)e^{i(\alpha_2+\alpha_3)p.\tilde{q}}}{\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2}+ \\
&\quad+\frac{m\alpha_1(\alpha_2+\alpha_3)\left(1-e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'}\right)}{m^2(\alpha_1-\alpha_2-\alpha_3)+m_\gamma^2(\alpha_2+\alpha_3)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2}\Bigg\},
\end{aligned}$$

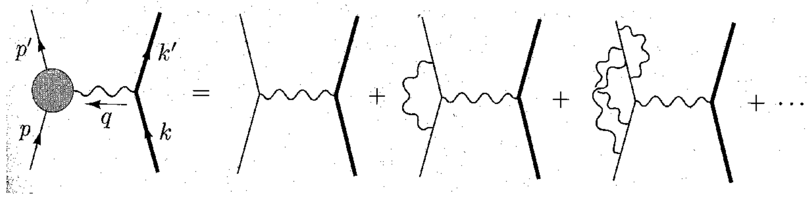


Figure 2

$$
\begin{aligned}
G_1 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi} im\gamma_{\text{Euler}} \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum \alpha_i\right) \times \\
&\quad \times \left((1+\alpha_2+\alpha_3)e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}} - (2-\alpha_2-\alpha_3)e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'}\right), \\
E_2 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi} (i\gamma_{\text{Euler}}) \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum \alpha_i\right) \times \\
&\qquad\qquad \times \left(1-e^{-ip\times p'}\right)(2-\alpha_2-\alpha_3)e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}} \\
H_3 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi} \frac{i\gamma_{\text{Euler}}}{2} \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum \alpha_i\right) \times \\
&\quad \times \left((2-\alpha_2-\alpha_3)e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}} + (1+\alpha_2+\alpha_3)e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}e^{-ip\times p'}\right). \qquad (3.28)
\end{aligned}
$$

In the next section we will study the physical significance of these factors.

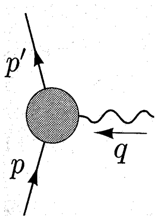
The only comment we would like to make in this part is the appearance of extra factors of $e^{-ip\times p'}$ in the above expressions. More precisely, our vertex function in the one loop level contains two different terms, one proportional to $e^{\frac{i}{2}p\times p'}$ and the other proportional to $e^{-\frac{i}{2}p\times p'}$. This can be understood if we note that our thoery is not CP invariant and if we change the arrow of time in our diagrams we will find the same expression, but we also should change θ to $-\theta$ [14].

4. The electron-photon vertex function

In this section we give a formal discussion of the electron-photon vertex structure found in NCQED. So, first we review the similar argument in the usual QED. The physical meaning of the different terms appearing in our vertex from the calculation in the previous section will be discussed later.

4.1 Electron-photon vertex in usual QED

In this section we briefly present the structure of the electron-photon vertex in the usual QED. Considering the radiative corrections to the vertex, it can be written in the following a diagrammatic form where the shaded circle can be expressed as



$$= -ie\bar{U}(p')\Gamma^{\mu}(p',p)U(p)\tilde{A}_{\mu}(q)\,. \tag{4.1}$$

We observe that in the leading order of perturbation $\Gamma^{\mu} \to \gamma^{\mu}$.

The list of available vectors and scalars is only consisting of $(p_{\mu}, p'_{\mu}, \gamma_{\mu}, q^2, m, e)$. So, the most general form of the vertex function can be written as [4]

$$\Gamma^{\mu} = A\gamma^{\mu} + B(p'+p)^{\mu} + C(p'-p)^{\mu}\,, \tag{4.2}$$

where A, B, C can be any scalars formed out of previous list. Appealing to Ward identity i.e. $q^{\mu}\Gamma_{\mu} = 0$, we get

$$0 = A\,\not{q} + B(p'+p)\cdot q + C(p'-p)\cdot q\,. \tag{4.3}$$

Since, $\bar{U}(p')\,\not{q}U(p) = 0$, $(p'+p)\cdot q = 0$ and $(p'-p)\cdot q \neq 0$, we conclude that to satisfy the Ward identity, $C = 0$. Therefore, Γ^{μ} for QED takes the following form,

$$\Gamma^{\mu} = A\gamma^{\mu} + B(p'+p)^{\mu}\,, \tag{4.4}$$

using the Gordon identity we can write the above expression as,

$$\Gamma^{\mu} = F_1(q^2)\gamma^{\mu} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\,. \tag{4.5}$$

$F_1(q^2)$ and $F_2(q^2)$ which usually are known as form factors, are functions of m, e and q^2 depending on the diagram. One can see that at the tree level we have $\Gamma^{\mu} \to \gamma^{\mu}$, hence $F_1(q^2) = 1$ and $F_2(q^2) = 0$.

To give an interpretation to the above form factors we will couple this vertex to external slowly varying electric and magnetic fields at the tree level.

The electron electric charge. Let us consider the Coulomb scattering cross section, from a slowly varying electric field, i.e. we can take the limit $q^2 = 0$. The corresponding scattering amplitude can be written as

$$-iM = -ieF_1(0)\tilde{\Phi}(q)\xi^{\dagger}\xi\,, \tag{4.6}$$

where we have used the non-relativistic approximation for the spinor field,

$$U(p) = \frac{1}{\sqrt{2}}\begin{pmatrix}\left(1 - \frac{\vec{p}\cdot\vec{\sigma}}{2m}\right)\xi \\ \left(1 + \frac{\vec{p}\cdot\vec{\sigma}}{2m}\right)\xi\end{pmatrix}. \tag{4.7}$$

In the Born approximation $V(x) = eF_1(0)\Phi(x)$, hence we can identify $F_1(0)$ as the electric charge in units of e, and since $F_1(0) = 1$ at the tree level, radiative corrections to $F_1(q^2)$ should vanish at $q^2 = 0$.

[4] We have not included γ^5 because of the parity conservation.

The electron magnetic moment. We now repeat the analysis for an electron scattering from a static vector potential. The amplitude of scattering from this field is

$$iM = ie\left[\bar{U}(p')\left(\gamma^i F_1 + \frac{i\sigma^{i\nu}}{2m}q_\nu F_2\right)U(p)\right]\tilde{A}^i(q)\,. \tag{4.8}$$

Inserting the non-relativistic expansion for the spinors (4.7), and keeping terms first order in momenta we obtain,

$$iM = -ie\xi^\dagger\left(\frac{-\sigma_k}{2m}[F_1(0)+F_2(0)]\right)\xi\tilde{B}^k(q)\,, \tag{4.9}$$

where

$$\tilde{B}^k(q) = -i\epsilon^{ijk}q_i\tilde{A}_j(q)\,, \tag{4.10}$$

is the Fourier transform of the magnetic field produced by $A(x)$.

Again we interpret M as the Born approximation to the scattering of the electron from a potential. The potential is just that of a magnetic moment interaction, $V(x) = -\langle\mu\rangle\cdot B(x)$, where,

$$\langle\vec{\mu}\rangle = \frac{e}{m}[F_1(0)+F_2(0)]\xi^\dagger\frac{\vec{\sigma}}{2}\xi\,. \tag{4.11}$$

This expression for the magnetic moment of the electron can be rewritten in the standard form

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}\,, \tag{4.12}$$

where $\vec{S}$ is the electron spin. The coefficient g is

$$g = 2[F_1(0)+F_2(0)] = 2 + 2F_2(0)\,. \tag{4.13}$$

Along our previous argument for the leading order of F_1 and F_2, we see that to the leading order (classical level) the magnetic moment of a Dirac particle is 2.

4.2 Dipole moment at the tree level in NCQED

The calculation of the vertex function at the tree level goes exactly as in the case of usual QED, except for the extra phase $e^{\frac{i}{2}p\times p'}$ appearing due to the star product, i.e.

$$\begin{aligned}\Gamma^\mu &= e^{\frac{i}{2}p\times p'}\gamma^\mu\,,\\ &= e^{\frac{i}{2}p\cdot\tilde{q}}\gamma^\mu\,.\end{aligned} \tag{4.14}$$

Using a power expansion of the exponential and keeping only the first two terms, we see that the first term in the expansion which is θ independent gives rise to the usual result in QED, i.e. we get the same electric charge and magnetic dipole moment. The second term which is proportional to θ will give rise to an *electric dipole moment* and couples to the external electric field, E, as $\langle P\rangle.E$, where

$$\langle P_i\rangle = \frac{1}{2}e\tilde{p}_i = \frac{1}{2}e\theta_{ji}p_j\,. \tag{4.15}$$

This term will also contribute to higher-pole moments when coupled to an external electro-magnetic field.

4.3 The electron-photon vertex structure in NCQED

In the above we discussed the structure of the electron-photon vertex, in the case of usual QED. The list of vectors and scaler appearing in the vertex function was restricted to $(\gamma^\mu, (p'+p)^\mu, q^2, m, e)$. In the case of NCQED, and due to the presence of $\theta^{\mu\nu}$, our previous list of independent vectors and scalars will be extended to include three other scalars

$$\left(q^2, m, e, \tilde{q}\cdot\tilde{q}, \gamma\cdot\tilde{q}, p\cdot\tilde{q}\right),$$

and the list for the vectors will be

$$(\gamma^\mu, (p'+p)^\mu, q^\mu, \tilde{q}^\mu).$$

The most general structure of the vertex function which is compatible with the Ward identity, is

$$\Gamma^\mu = E\gamma^\mu + H(p'+p)^\mu + G\tilde{q}^\mu, \tag{4.16}$$

where E, F and G are scalars formed from the list of our previous scalars, except for $\tilde{q}.\tilde{q}$, which as discussed in the previous section will not appear in the first loop. Up to the one loop approximation the coefficients E, F and G may be expanded to be written as functions of $q^2, p\cdot\tilde{q}$ and $\gamma\cdot\tilde{q}$. The form factors $F_1(q^2)$ and $F_2(q^2)$ can be picked directly from E_1 and H_1 using the Gordon identity. From the results of the previous calculations one can see that the only non-zero coefficients are E_1, E_2, H_1, H_3 and G_1, i.e.

$$\Gamma^\mu = E_1\gamma^\mu + H_1(p'+p)^\mu + G_1\tilde{q}^\mu + E_2\gamma^\mu p\cdot\tilde{q} + H_3(p'+p)^\mu\gamma\cdot\tilde{q}. \tag{4.17}$$

So what is left now is to give a physical interpretation to those coefficients proportional to θ. We will use the non-relativistic limit, to compute $\bar{U}(p')\Gamma^\mu U(p)$. Keeping terms up to second order of momentum we find,

- G_1, coefficients of $\tilde{q}_\mu$:

  This term will give a contribution to the magnetic moment. The corresponding effective interaction potential with the external magnetic filed is $V(x) = -\langle\mu\rangle.B(x)$, where

  $$\langle\vec{\mu}\rangle = \frac{G_1}{i}\vec{\theta}, \qquad \theta_i \equiv \epsilon_{ijk}\theta_{jk}. \tag{4.18}$$

  As we see this magnetic moment does not depend on spin.

- E_2, coefficients of $\gamma^\mu p.\tilde{q}$:

  This term will vanish in the non-relativistic limit, but it will give contribution to higher moments when higher orders of momentum are considered.

- H_3, coefficients of $(p'+p)^\mu \gamma.\tilde{q}$:

  This term will give rise to an electric dipole moment of the form

$$\langle P\rangle_i = 2iH_3\tilde{p}_i\,.$$

Since we are going to work in the low momentum limit, we can use the series expansion of $e^{-i(\alpha_2+\alpha_3)p\cdot\tilde{q}}$, $e^{i(\alpha_2+\alpha_3)p\cdot\tilde{q}}$ and $e^{ip'\times p}$ and keep only the first term of the expansion. In this limit

$$\begin{aligned}
E_1 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)\times\\
&\times\left[\frac{(2p'\cdot p-(\alpha_2+\alpha_3)(p'+p)^2)+m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3 q^2)}{2(\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2)}+\gamma_{\mathrm{Euler}}\right],\\
H_1 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)\frac{m\alpha_1(\alpha_2+\alpha_3)}{\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2},\\
G_1 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}im\gamma_{\mathrm{Euler}}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)(\alpha_2+\alpha_3-1)\,,\\
H_3 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\frac{3i\gamma_{\mathrm{Euler}}}{2}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right).
\end{aligned}\tag{4.19}$$

Now we can apply the Gordon identity to E_1 and H_1, and identify the form factors $F_1(q^2)$ and $F_2(q^2)$ directly

$$\begin{aligned}
F_1(q^2) &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)\times \qquad (4.20)\\
&\times\left[\frac{2m^2(1-\alpha_2-\alpha_3)-q^2(1-\alpha_2-\alpha_3)-m^2(\alpha_2+\alpha_3)^2-\alpha_2\alpha_3q^2}{2(\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2)}+\gamma_{\mathrm{Euler}}\right],
\end{aligned}$$

and

$$F_2(q^2) = \frac{\alpha e^{\frac{i}{2}p\times p'}}{\pi}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\left(1-\sum\alpha_i\right)\frac{m^2\alpha_1(\alpha_2+\alpha_3)}{\alpha_1 m_\gamma^2+(\alpha_2+\alpha_3)^2m^2-\alpha_2\alpha_3q^2}.\tag{4.21}$$

As it is seen, apart from the pre-factor, $e^{\frac{i}{2}p\times p'}$, the (4.20), (4.21) expressions are exactly the same as the usual QED.

The integration over the Schwinger parameters in G_1, and H_3 terms can be easily performed,

$$\begin{aligned}
G_1 &= \frac{\alpha e^{\frac{i}{2}p\times p'}}{\pi}\left(\frac{im\gamma_{\mathrm{Euler}}}{6}\right),\\
H_3 &= \frac{-\alpha e^{\frac{i}{2}p\times p'}}{\pi}\left(\frac{3i\gamma_{\mathrm{Euler}}}{4}\right).
\end{aligned}\tag{4.22}$$

Altogether, we can write the interaction of a noncommutative electron and an external electromagnetic field in the form,

$$V(x) = e\Phi + \langle\mu\rangle \cdot B + \langle P\rangle \cdot E\,, \tag{4.23}$$

where the first term is just a Coulomb potential, the second is the magnetic dipole moment and the last term is the electric dipole moment. The coefficients $\langle\mu\rangle$ and $\langle P\rangle$, in the low momentum approximation, and up to first loop are

$$\begin{aligned}\langle\vec{\mu}\rangle &= \frac{e}{m}\left[(F_1(0)+F_2(0))\vec{S} + \frac{\alpha\gamma_{\rm Euler}}{6\pi}m^2\vec{\theta}\right],\\ \langle\vec{P}\rangle &= \frac{1}{4}e(\vec{\theta}\times\vec{p})\,\left(1+3\alpha\gamma_{\rm Euler}\right)\end{aligned} \tag{4.24}$$

5. Concluding remarks

In this work we have discussed some aspects of NCQED. First we introduced the theory by giving its action, and the corresponding basic Feynman graphs. We argued that the photon itself, similar to the moving noncommutative electron, even at classical level, shows some electric dipole effect. However, the dipole moment of the fermions compared to that of photon, is less by a factor of one half. This is due to the fact that in the NCYM action, the noncommutative effects appear as the Moyal-bracket, while in the fermionic sector it is just a star product.

Calculating the one loop contributions to the electron-photon vertex, we studied the electro-magnetic dipole moments of the electron. We showed that magnetic dipole moment of electron has now two parts, one spin dependent which will not receive any further corrections due to the noncommutativity, and the other spin independent, being proportional to θ. In addition we also found the one loop contributions to the electric dipole moment.

Here we only studied the loop effects coming form the interaction vertex correction, however there are many other interesting phenomena coming from the vacuum polarization and corrections to propagators such as, Lamb shift, or the pair production threshold, which we postpone them to the future studies.

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A. Some useful identities in $*$-product calculus

Let f, g be two arbitrary functions on non-commutative R^d:

$$f(x) = \int f(k) e^{ik\cdot x} d^d k\,, \qquad g(x) = \int g(k) e^{ik\cdot x} d^d k\,.$$

Then

$$(f * g)(x) = \int f(k) g(l) e^{-ik\theta l/2} e^{i(k+l)\cdot x} d^d k d^d l\,,$$

where $k\theta l = k^\mu \theta_{\mu\nu} l^\nu$. From the above relation it is straightforward to see:

1. $g * f = f * g|_{\theta\to -\theta}$, and hence $\{f, g\}_{M.B.} = f * g|_\theta - f * g|_{-\theta}$.
2. $\int (f * g)(x) d^d x = \int (g * f)(x) d^d x = \int f g(x) d^d x$.
3. If we denote complex conjugation by c.c., then $(f * g)^{\text{c.c.}} = g^{\text{c.c.}} * f^{\text{c.c.}}$.

If h is another arbitrary function:

4. $(f * g) * h = f * (g * h) \equiv f * g * h$.
5. $\int (f * g * h)(x) d^d x = \int (h * f * g)(x) d^d x = \int (g * h * f)(x) d^d x$.
6. $(f * g * h)|_\theta = (h * g * f)|_{-\theta}$.

In other words the integration on the space coordinates, x, has the cyclic property, and it has all the properties of the Tr in the matrix calculus.

From 2) we learn that the kinetic part of the actions (which are quadratic in fields) is the same as their commutative version. So the free field propagators in commutative and nnoncommutative spaces are the same.

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